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## No. 20

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A new method for the numerical integration presented for the first time in this paper is applied to the following surfaces: flat and curved rectangular, cylindrical, conical, spherical, toroidal, and elliptical. Results are compared with those obtained by the Simpson's rule, which is well known for its accuracy. In addition, the results are compared with those obtained by the Gauss-Legendre quadrature, which is known to be the most accurate. The results show that the proposed method is more accurate than the other two methods. The proposed method is applied to the calculation of the capacitance of a cylindrical capacitor. The results are compared with those obtained by the Gauss-Legendre quadrature. The results show that the proposed method is more accurate than the other two methods. The proposed method is applied to the calculation of the capacitance of a cylindrical capacitor. The results are compared with those obtained by the Gauss-Legendre quadrature. The results show that the proposed method is more accurate than the other two methods.

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$(\mathbf{e}_r, \mathbf{e}_\theta)$	Unit base vectors in a cylindrical polar reference system
$(\mathbf{e}_x, \mathbf{e}_y)$	Unit base vectors in a Cartesian reference frame
$\Gamma_\infty = V_\infty b$	A Kármán coefficient based on reference speed
$\mathbf{N} = (\mathbf{x} \times \mathbf{y})/r$	Vector normal to blade surface pointing into fluid
$N_R = (\mathbf{x} \times \mathbf{y})/r$	Radial component of $\mathbf{N}$
$N_\infty = (\mathbf{x} \times \mathbf{y})/r$	Normal to blade reference surface ( $r = 0$ ) surface
$\mathbf{n}$	Unit vector normal to blade surface pointing into fluid
$\Omega$	Propeller rotational speed (revolutions per unit time)
$p$	Pressure
$P(\alpha_R)$	Pitch of blade section
$Q$	Torque absorbed by blades
$\mathbf{u} = (u, v, w)$	Velocity vector
$\mathbf{u}_\infty$	Velocity vector far upstream
$R = D/2$	Rotor tip radius
$r_h$	Radius of rotor hub
$\mathbf{r}$	Position vector of field point
$\mathbf{r}_b$	Position vector of field point on blade reference surface
$\mathbf{r}_b^k(x, y, \mathbf{R})$	Position vector of point on $k^{\text{th}}$ blade surface
$\mathbf{r}_b(x, y, \mathbf{R})$	Position vector of point on blade reference surface $\theta_b = 0$
$\mathbf{r}_{w_b}(\eta, \mathbf{R})$	Position vector of point on shed vortex sheet
$T$	Thrust produced by blades
$t$	Thickness of blade section
$V$	Reference speed
$v$	Velocity component due to presence of the blades
$\mathbf{v} = (u, v, w)$	Average perturbation velocity along blade surface due to presence of the blades
$\mathbf{v} = \mathbf{v}^* + (\gamma/\mu)\mathbf{n}$	Velocity difference across blade surface
$\mathbf{v}_1$	Even perturbation velocity component due to blade loading and shed vortex sheet

$v_T$	Even perturbation velocity component due to thickness	$\mu(x_L, x_R)$	Normal component of disturbance velocity difference across blade section (source strength due to thickness)
$w$	Velocity induced by vortex filament	$(\xi_1, \xi_2, t)$	Helical coordinates on pitch reference surface
$w_{\theta}(x_R)$	Local wake traction	$\rho$	Fluid density
$w_R(x_R)$	Radial free stream velocity component (traction of $\Lambda$ )	$\sigma(x_L, x_R)$	Component of disturbance velocity difference across blade section
$(x, y, z)$	Cartesian coordinates	$\sigma_A$	Surface area
$x$	Cartesian coordinate for field point on blade surface	$\phi$	Potential function for perturbation velocity, polar coordinate for field point
$x_c$	Fraction of chord measured from leading edge	$\phi_P(x_R) = \tan^{-1}(P/D) / \pi x_R$	Pitch angle of blade reference surface, measured on cylinder of radius $r$
$x_L$	Fraction of chord for field point on blade surface	$\phi_R(x_R)$	Geometric pitch angle
$x_h$	Hub radius, fraction of tip radius	$\psi(x_L)$	Radius of streamline on blade surface
$x_R$	Fraction of radius measured from axis of rotation	$\omega$	Angular variable in radial direction
$x_R$	Radial coordinate for field point on blade surface		
$Y_T(x)$	Nondimensional thickness offset maximum $Y_T = 0.5$		
$Z$	Number of blades		
$\alpha$	Angular variable in chordwise direction		
$\alpha(x_L, x_R)$	Component of derivative of surface coordinate		
$\beta = \tan^{-1} \frac{t(x) - w_{\theta}(x)}{x_R \pi}$	Advance angle of blade section		
$\Gamma(x_R)$	Circulation distribution		
$\gamma(x_L, x_R)$	Chordwise component of disturbance velocity difference across blade section		
$\gamma^*(x_L)$	Chordwise velocity difference scaled to give unit magnitude when integrated across the chord		
$\epsilon$	Error bound, increment to pitch angle when radial inflow exists		
$\eta$	Integration variable along vortex filament		
$\theta = \tan^{-1} \frac{y}{z}$	Angular coordinate in cylindrical reference frame		
$\theta_b = 2\pi(b-1)/Z$	Angular coordinate of blade reference line of $b$ th blade		
$\theta_s(x_R)$	Skew angle, circumferential displacement of blade-section mid-chord point from $y = 0$ plane		
$\theta_{\phi}(x_L, x_R)$	Angular coordinate of point on blade reference surface		
$\Lambda = (\sigma, \gamma)$	Vorticity vector		

## INTRODUCTION

The design of an open marine propulsor is a complex process, involving structural and hydrodynamic considerations (1, 2). For the hydrodynamic considerations during most of the preliminary design process, approximate models of the lifting surfaces are employed, e.g., the lifting-line model (3, 4) for powering considerations, and two-dimensional flow over equivalent blade sections for cavitation performance. More sophisticated models of the lifting surfaces are used for predicting fluctuating loads (5) and some cavitation predictions (6). These approximate models have been acceptable during the preliminary design process and provide a basis for choice of the maximum diameter, advance coefficient and radial variations of chord, skew-angle, rake, thickness, and circulation distribution. The chordwise variation in load has usually been selected during this preliminary stage and is often based on cavitation and propulsion considerations.

For the final stage of the design, the meanline distribution and radial pitch variation are determined corresponding to the selections for load and geometry already available. To derive a geometry which accurately produces the specified load distributions, a lifting-surface model of the blades is required.

Several procedures already exist for performing lifting-surface calculations for wide-bladed open marine propulsors. In particular, two different approaches to the analysis for blades with arbitrary locations in space have been presented by Kerwin (7) and McMahon (8). Kerwin's numerical analysis procedure is based on three fundamental assumptions: (1) that the continuous loading distribution on the nonplanar blade surface can be adequately approximated by a multitude of discrete straight lines of constant-vortex strength and that the source distribution arising from the thickness distribution can be similarly approximated, (2) that the minimum required spacing between lattice elements along the chordline is  $\Delta\theta = 2$  degrees, and (3) that the resulting meanline shape for a given chordwise load is similar to the two-dimensional shape for the same chordwise load. The first two assumptions are not acceptable for very narrow blades— for a blade with a 20 degree pitch angle at the 0.9



radius and a chord to diameter ratio of 0.05, the 2 degree spacing equals increments of about 1/3 chord length. The last assumption permits calculations to be performed using only a few points along the chord and the two-dimensional shape is fitted to the data at these points. The resulting computer code is relatively quick running and produces a geometry which, in practice, has an overall speed and power performance generally within a few percent or so of the predicted values, with a general tendency to produce a greater thrust than predicted. The procedure of McMahon employs continuous distributions for the loading and thickness functions and calculates the meanline from the induced velocity. Consequently, data at more chordwise points are required to define the pitch and meanline distributions. The resulting computer code is lengthy to run but has shown remarkably different meanline shapes from the two-dimensional one at the hub and tip region of the blade where the meanlines can be shaped (8). Two models were constructed and experimentally evaluated to provide data on the relative cavitation and propulsion performance of designs having the same input specifications but final geometry according to the Kerwin and McMahon procedures. Some inconsistencies occurred in the experimental measurements but the thrust was closer to the predicted value and the operating point centered in the cavitation bucket for the model designed by the McMahon method. Hence, the determination of specific meanline and pitch distributions, instead of fitting the two-dimensional meanline, is considered to be a superior procedure when the design is based on a narrow range of permissible operating conditions and the delay of cavitation is critical.

Because the numerical-analysis procedure employed by McMahon results in lengthy computer runs and Kerwin's procedure is not acceptable for narrow blades, alternative numerical-analysis schemes are investigated in this paper. In addition, a detailed description of the flow field across the blade surface was desired as input into boundary-layer calculations. Two different numerical analysis schemes are described, each involving an expansion of the singular kernel about the singular point. Both approaches employ integration of the specified thickness slope and load distribution over the reference blade in the radial direction first and the remaining chordwise integration then takes the form of the velocity component corresponding to two-dimensional flow modified by the presence of an induction factor in the integral. Regular integration techniques are employed for the other blades and the shed vortex sheet. The induced velocity components are appropriately combined and integrated to obtain meanline shapes.

The present investigation describes the real-fluid flow about a rotating system of lifting surfaces having both loading and thickness. Several approximations are made. The first of these is the mathematical model for which potential flow equations are employed and the solution to first-order in thickness-to-chord ratio, camber-to-chord ratio and difference in pitch and flow angles derived. Comparisons with experimental results for other lifting-surface configurations lead to confidence in this linearized approximation. In addition to this mathematical model, further approximations occur in the numerical analysis. Confidence in the numerical analysis procedures is justified by comparison with analytical solutions or experimental results. That is, results are sought from some discretized numerical-analysis procedures involving  $N$  by  $M$  approximations, which have converged to within some specified tolerance,  $\epsilon$ , of the real or analytical value of the quantity investigated. Mathematically this may be stated

$$|f(x, y) - f_{N, M}(x, y)| < \epsilon$$

$$\text{for } \begin{cases} (x, y) \text{ on the surface } S \\ N \geq N_0 \\ M \geq M_0 \end{cases}$$

where  $f_{N, M}$  = the approximate calculation of a particular quantity  $f$

$S$  = a region of the surface of interest

$N_0, M_0$  = minimum numbers of the discrete approximations for which the computed results are within  $\epsilon$  of the values for  $f$

For rotating lifting surfaces, neither measured nor analytical solutions exist for details of the flow field on the blade. Hence, comparisons will be made with other procedures. It is assumed that numerical solutions which employ increasingly greater pointwise definition of the input variable without change in computed values have converged and that the solution has converged when a smooth curve can be drawn through point values in both the chordwise and radial directions. These assumptions are believed to be necessary but not sufficient for convergence.

In the following sections, the mathematical model of the flow field on the blade surface is first reviewed and numerical-analysis techniques for evaluating both regular and singular integrals are described. A FORTRAN computer code is discussed and sample calculations using this code are presented. From example calculations, it is found that greater accuracy in the integral evaluations is required for the determination of smooth pressure distribution curves than for the shape of the meanline and the pitch distributions. The choice of a particular chordwise loading distribution is shown to have an effect on the meanline shape and the pressure distribution. The effects of rake and skew are shown to be important on both pressure distribution and meanline shape. A particular thickness function has hardly any effect on pitch or meanline but a significant effect on pressure distribution.

## MATHEMATICAL MODEL THICK LIFTING BLADE

The mathematical model of a system of rotating lifting surfaces advancing in an unbounded irrotational flow field with an inviscid fluid has been developed on a formal mathematical basis by Brockett (9). A reformulation of that analysis in terms of non-dimensional surface coordinates is presented herein for completeness. The propulsor is assumed to be adequately represented by the blades alone, i.e., neither the hub nor fillet from the blades to the hub is included in the blade specification. The onset flow is assumed to be directed along the axis of rotation but a new feature included herein is that it may have a small radial component. Overall geometry notation generally follows the definitions given in Reference 10.

Coordinate systems are constructed with the same orientation as in Reference 9, and in particular, the helical coordinate system  $(\xi_1, \xi_2, r)$  rotating with the blades is shown in Figure 1. Unit base vectors in a right-handed Cartesian reference frame are the customary  $(i, j, k)$  where  $i$  is along the  $x$  axis and is positive pointing aft,  $j$  is along the  $y$  axis and  $k$  is along the  $z$  axis which is generally along the reference blade. Unit vectors along the helical coordinates are

$$e_1 = \sin \phi_p i + \cos \phi_p e_R \quad (1)$$

$$e_2 = \cos \phi_p i + \sin \phi_p e_R \quad (2)$$

$$e_r = -\sin \theta j + \cos \theta k \quad (3)$$

where

$$e_R = \cos \theta j + \sin \theta k \quad (4)$$



$$q = V(1 + w_R(r/R)) + 2\pi n r c_R \quad (12)$$

$$+ V w_R(r/R) c_r + v$$

$$q_{ax} = v \quad (13)$$

where  $V$  — the constant reference speed

$1 + w_R$  — the wake fraction multiple to obtain the local axisymmetric speed<sup>1)</sup>

$w_R$  — the radial component of inflow fraction of the reference speed

$n$  — the rotational speed, revolutions per unit time, and

$v$  — the velocity component due to the presence of the blades

If  $2p$  is the pitch angle and  $\beta$  is the advance angle

$$q_r = \tan \beta \sqrt{V^2 + w_R^2} + 2\pi n r c_r$$

then

$$\frac{q_r}{V} = \sqrt{1 + w_R^2} + \left(\frac{2\pi n r}{V}\right) \quad (14)$$

$$\int_0^1 \sqrt{1 + w_R^2} dr = \left(1 + \frac{1}{2} w_R^2\right) + \frac{1}{4} w_R^4$$

where the advance coefficient  $\beta$  is given by

$$\tan \beta = V \cos \beta$$

The velocity  $q_r$  is the resultant of the tangential velocity  $q$  and the axial velocity  $v$

$$N = \sqrt{q^2 + v^2} = \sqrt{q^2 + q_r^2} \quad (15)$$

The velocity  $q$  is the resultant of the tangential velocity  $q$  and the radial velocity  $w_R$

$$q = \sqrt{V^2 + w_R^2}$$

$$N = \sqrt{V^2 + w_R^2 + v^2}$$

The velocity  $v$  is the resultant of the tangential velocity  $v$  and the axial velocity  $v$

$$v = \sqrt{V^2 + w_R^2} + 2\pi n r c_r$$

Now

$$N = \sqrt{V^2 + w_R^2 + v^2}$$

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The difference of Equations (12) and (13) gives

$$q_r = N^2 - V^2 - w_R^2 - 2\pi n r c_r = N^2 - N^2$$

$$q_r = \left[ N^2 - V^2 - \frac{1}{2} \frac{D}{\rho V^2} \left( \frac{1}{2} \frac{D}{\rho V^2} \right) \right]$$

$$\left( \left[ N^2 - V^2 - \frac{1}{2} \frac{D}{\rho V^2} \left( \frac{1}{2} \frac{D}{\rho V^2} \right) \right] \right)$$

$$q_r = \sqrt{V^2 + w_R^2}$$

$$D = V \left[ \sqrt{V^2 + w_R^2} + \left( \frac{2\pi n r}{V} \right) \right]$$

$$\left( \frac{1}{D} \left( \frac{1}{2} \frac{D}{\rho V^2} \right) \left( \frac{1}{2} \frac{D}{\rho V^2} \right) \right)$$

$$\left[ \frac{1}{D} \left( \frac{1}{2} \frac{D}{\rho V^2} \right) \right] = \left( \frac{1}{D} \right) \left( \frac{1}{2} \frac{D}{\rho V^2} \right)$$

$$\left[ \frac{1}{D} \left( \frac{1}{2} \frac{D}{\rho V^2} \right) \right] = \left( \frac{1}{D} \right) \left( \frac{1}{2} \frac{D}{\rho V^2} \right)$$

The velocity  $q$  is the resultant of the tangential velocity  $q$  and the radial velocity  $w_R$

$$q = \sqrt{V^2 + w_R^2} + 2\pi n r c_r$$

$$\sum \theta$$

$$s_b = s_{ob} + D \left[ \left( \frac{1}{D} + \frac{1}{D} (x_c - 0.5) \sin \phi_p \right) + \frac{s_R}{2} e_r(\theta_{ob}) \right] \quad (25)$$

Hence, Equation (23) can be reduced to an integral over only one side of the blade surfaces and shed vortex sheets

$$\begin{aligned} v(r) &= \frac{1}{4\pi} \sum_{b=1}^Z \int_0^1 \int_{x_h}^1 d x_c \int_{x_h}^1 d x_R \\ &\quad \cdot \left[ (N^+ + v^+ + N^- + v^-) \frac{r - s_{ob}}{|r - s_{ob}|^3} \right. \\ &\quad \left. + (N^+ + v^+ + N^- + v^-) \frac{r - s_{ob}}{|r - s_{ob}|^3} \right] \\ &\quad + \frac{1}{4\pi} \sum_{b=1}^Z \int_{x_h}^1 \int_0^\infty \frac{d x_c}{d\eta} [N^+ + (v^+ - v^-)] \\ &\quad \cdot \frac{r - s_{ob}}{|r - s_{ob}|^3} \quad (26) \end{aligned}$$

where

$$\begin{aligned} N_{ob}^+ &= \left( \frac{s_{ob}}{D} + s_{ob} \sin \phi_p \right) \\ &\quad + \frac{s_R}{2} e_r(\theta_{ob}) + s_{ob} \cos \phi_p \cdot s_R + a_{b1} \end{aligned}$$

and

$$s_{ob} = s_{ob}(r) \quad (27)$$

As the field point  $r$  approaches a point  $r_o$  ( $x_{co} = x_{Ro}$ ) on the surface of the blade, Equation (24) or (26) becomes singular. If a small region about this point is excluded from the surface  $S$ , and the limit of the integral taken for  $r \rightarrow r_o$  with the excluded area tending to zero, there results the kernel  $K(r_o)$

$$\begin{aligned} K(r_o) &= \frac{1}{4\pi} \sum_{b=1}^Z \int_0^1 \int_{x_h}^1 d x_c \int_{x_h}^1 d x_R \\ &\quad \cdot \left[ (N^+ + v^+ + N^- + v^-) \frac{r_o - s_{ob}}{|r_o - s_{ob}|^3} \right. \\ &\quad \left. + (N^+ + v^+ + N^- + v^-) \frac{r_o - s_{ob}}{|r_o - s_{ob}|^3} \right] \\ &\quad + \frac{1}{4\pi} \sum_{b=1}^Z \int_{x_h}^1 \int_0^\infty \frac{d x_c}{d\eta} [N^+ + (v^+ - v^-)] \\ &\quad \cdot \frac{r_o - s_{ob}}{|r_o - s_{ob}|^3} \quad (28) \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{4\pi} \sum_{b=1}^Z \int_{x_h}^1 \int_0^\infty \frac{d x_c}{d\eta} \frac{d x_R}{d\eta} \\ &\quad \cdot \frac{[N^+ + (v^+ - v^-)] \cdot (r_o - s_{ob})}{|r_o - s_{ob}|^3} \end{aligned}$$

where the symbol  $\oint$  means symmetry restriction on  $r$  to the limiting region which excludes the singularity. For example (9), the region may be square (circular or rectangular) centered at  $r_o$ . In the present application, the rectangular region  $x_{co} - \epsilon < x_c < x_{co} + \epsilon$ ,  $x_h - \epsilon < x_R < x_h + \epsilon$  will be the shape of the excluded region. Then this principal value integral is defined

$$\begin{aligned} \oint d x_c \int_{x_h}^1 d x_R \cdot K &= \lim_{\epsilon \rightarrow 0} \left[ \int_0^{x_{co}-\epsilon} \int_{x_h}^1 d x_c \int_{x_h}^1 d x_R \cdot K \right. \\ &\quad \left. + \int_{x_{co}+\epsilon}^1 \int_{x_h}^1 d x_c \int_{x_h}^1 d x_R \cdot K \right] \quad (29) \end{aligned}$$

The assumption

$$\lim_{r \rightarrow r_o} [v(r)] = v^+(r_o) \quad (30)$$

(i.e., that the velocity defined in the field does approach the value on the boundary) leads to the following expression for the average velocity component on the blade surface

$$v(x_{co}, x_{Ro}) = \frac{1}{2} \left[ v^+(x_{co}, x_{Ro}) + v^-(x_{co}, x_{Ro}) \right] \quad (31)$$

$$= v_1 + v_2$$

$$= u e_1 + v e_2 + w e_3$$

$$= \sum_{b=1}^Z \oint d x_c \int_{x_h}^1 d x_R \cdot K(x_{co}, x_{Ro}, x_c, x_R) + v_w \quad (32)$$

where the singular kernel is

$$\begin{aligned} K(x_{co}, x_{Ro}, x_c, x_R) &= \frac{1}{4\pi} \left[ (N^+ + v^+ + N^- + v^-) \frac{r_o - s_{ob}}{|r_o - s_{ob}|^3} \right. \\ &\quad \left. + (N^+ + v^+ + N^- + v^-) \frac{r_o - s_{ob}}{|r_o - s_{ob}|^3} \right. \\ &\quad \left. + \frac{r_o - s_{ob}}{|r_o - s_{ob}|^3} \right] \\ &\quad + \frac{1}{4\pi} \sum_{b=1}^Z \int_{x_h}^1 \int_0^\infty \frac{d x_c}{d\eta} [N^+ + (v^+ - v^-)] \\ &\quad \cdot \frac{r_o - s_{ob}}{|r_o - s_{ob}|^3} \end{aligned}$$

and the velocity induced by the shed vortex sheet is

$$-\frac{1}{2\pi} \sum_{j=1}^n \int_{\Gamma_j} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D}$$

$$N^* = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) N^* = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) N^*$$

where  $\Gamma_j$  is the strength of the sources and  $\Gamma_j$  is the strength of the vortices in pairs.

$$N^* = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) N^* = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) N^*$$

where  $\Gamma_j$  is the strength of the blades and  $\Gamma_j$  is the strength of the vortices in pairs. The value of  $\Gamma_j$  is found by applying the condition that the blade section is a free vortex.

$$\Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j$$

where  $\Gamma_j$  is the strength of the blades

$$\Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j$$

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$$\Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j$$

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$$\Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j$$

Let

$$\Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j$$

Then to first order

$$\Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j$$

$$\Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j$$

and from Equation (20)

$$\Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Gamma_j$$

where

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

It remains to determine  $\Delta C_p$  from general

$$N^* = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) N^* = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) N^*$$

and the value of  $\Delta C_p$  can be determined by setting the right-hand side equal to zero. However, this results in a differential equation to be solved for  $\Delta C_p$ . The first procedure is to express the perturbation velocity as the gradient of a potential

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

where

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

Thus

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

and

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

For  $\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

It is convenient to define

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

where  $\Gamma(x_R)$  is the bound circulation ( $\phi^* = \phi$ ) at the trailing edge and points beyond) and  $\gamma^*$  has unit magnitude when integrated across the chord. Let the nondimensional circulation  $G$  be

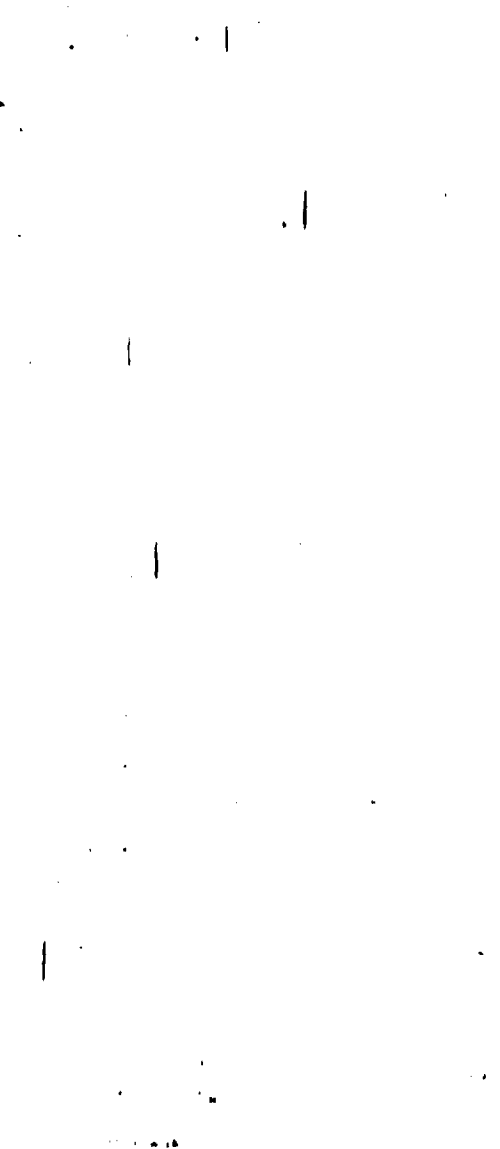
$$G = \frac{\Gamma}{\pi DV}$$

Then

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$

and

$$\Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p = \frac{1}{2\pi} \left( \frac{1}{D} \int_{\Gamma_j} \frac{d\mathbf{x}_j}{D} \right) \Delta C_p$$



the strength of the vortex distribution is explicit and the integration to determine the average in the elements of the blade surface can be undertaken

the meanline offset to field on the blade is given by the meanline slope can be determined and the meanline offset can be found by integrating the slope

$$y = \int_0^x \frac{dy}{dx} dx = \int_0^x \frac{1}{2} \frac{d}{dx} \left( \frac{N_a^2}{N_a^2} \right) dx \quad (60)$$

From Equation (60) the meanline offset for the term with the meanline velocity component can be directly computed. It consists of an angle of attack term due to gradient of the rake and skew terms and a parabolic arc meanline due to gradients of the pitch



An approximation for the non-linear speed of the blade in the chordwise direction is given by

$$\frac{dV}{dx} = \frac{V^2 + \left( \frac{w_R}{V} \right)^2 + \left( \frac{w_T}{V} \right)^2}{V + \left( \frac{d(F_T/V)}{dx} \right)^2} \quad (61)$$

Hence

$$\left( \frac{q}{V} \right)^2 = \left( \frac{q_T}{V} \right)^2 + \left( w_R e_T + \hat{e} + \frac{1}{V} + \hat{e} \right)^2 \quad (62)$$

where  $\hat{e}$  is the unit vector in the  $N_a^2 \times e_T$  direction (nearly the radial direction over much of the blade)

$$e = \frac{N_a^2 \times e_T}{|N_a^2 \times e_T|} = \frac{e_T + N_{R_a} e_T}{\sqrt{1 + N_{R_a}^2}} \quad (63)$$

[illegible]
$$\int_0^T \left( \|w\|_{L^\infty(\Omega)} + \|w\|_{L^6(\Omega)} \right) dt \leq C_T \left( \|f\|_{L^2(\Omega)} + \|g\|_{L^2(\Omega)} \right).$$

## NUMERICAL ANALYSIS FROM EQUATION (1)

value of the variable is the percentage of respondents that agree with the statement. Thus, the variable is computed as the mean of the six items. The composite reliability is 0.93, indicating that the scale is reliable and training is relevant. The Cronbach's alpha of the scale is 0.93, indicating that the scale is reliable. The composite reliability is 0.93, indicating that the scale is reliable and training is relevant. The Cronbach's alpha of the scale is 0.93, indicating that the scale is reliable.

On the top, we have the integral  $\int_{\mathbb{R}^d} \rho(x) dx$  and the bottom, the integral  $\int_{\mathbb{R}^d} \rho(x) dx$ . The top integral is the integral of the function  $\rho(x)$  over the whole space  $\mathbb{R}^d$ . The bottom integral is the integral of the function  $\rho(x)$  over the whole space  $\mathbb{R}^d$ .

$$\int \sum f$$

$$\sum$$

$$\sum$$

$$\left( \begin{matrix} n \\ k \end{matrix} \right) = \frac{n!}{k!(n-k)!}$$

$$\left( \begin{matrix} n \\ k \end{matrix} \right) = \frac{n!}{k!(n-k)!}$$

$$\sum$$



and  $A = \frac{1}{2} \left( \frac{c}{D} \right)^2$  and  $B = \frac{1}{2} \left( \frac{c}{D} \right)^2$ .

$$F(x_h, x_c) = \frac{1}{2} \left( \frac{c}{D} \right)^2 \left( x_h - x_c \right)^2 \left( 1 - \frac{c}{D} \right)$$

$$F(x_h, x_c) = \frac{1}{2} \left( \frac{c}{D} \right)^2 \left( x_h - x_c \right)^2 \left( 1 - \frac{c}{D} \right)$$

and

$$F(x_h, x_c) = \frac{1}{2} \left( \frac{c}{D} \right)^2 \left( x_h - x_c \right)^2 \left( 1 - \frac{c}{D} \right)$$

The linearized integrand  $F$  for integration over the blade reference surface then becomes in the general form where  $A$ ,  $B$ , and  $C$  depend on the particular case of loading or thickness:

$$F(x_h, x_c) = \sum_{i=1}^3 \int_{x_h}^{x_c} \frac{1}{\sqrt{A(x_h - x_{R_i})^2 + B(x_h - x_{R_i})(x_c - x_c) + \left(\frac{c}{D}\right)^2 (x_c - x_c)^2}} dx_{R_i} \quad (100)$$

$$F(x_h, x_c) = \sum_{i=1}^3 \int_{x_h}^{x_c} \frac{1}{\sqrt{A(x_h - x_{R_i})^2 + B(x_h - x_{R_i})(x_c - x_c) + \left(\frac{c}{D}\right)^2 (x_c - x_c)^2}} dx_{R_i}$$

where

$$D_i = \int_{x_h}^{x_c} \frac{(x_c - x_c)(x_h - x_{R_i}) dx_{R_i}}{\left[ \mu(x_h - x_{R_i})(x_c - x_c) \right]^2}$$

$$= \frac{2}{\left[ 4A \left( \frac{c}{D} \right)^2 - B^2 \right]} \quad (101)$$

$$F(x_h, x_c) = \frac{B(1 - x_{R_0}) + 2 \left( \frac{c}{D} \right)^2 (x_c - x_c)}{\sqrt{A(1 - x_{R_0})^2 + B(1 - x_{R_0})(x_c - x_c) + \left( \frac{c}{D} \right)^2 (x_c - x_c)^2}}$$

$$F(x_h, x_c) = \frac{1}{2} \left( \frac{c}{D} \right)^2 \left( x_h - x_c \right)^2 \left( 1 - \frac{c}{D} \right)$$

$$4A \left( \frac{c}{D} \right)^2 - B^2$$

$$F(x_h, x_c) = \frac{1}{2} \left( \frac{c}{D} \right)^2 \left( x_h - x_c \right)^2 \left( 1 - \frac{c}{D} \right)$$

$$F(x_h, x_c) = \frac{1}{2} \left( \frac{c}{D} \right)^2 \left( x_h - x_c \right)^2 \left( 1 - \frac{c}{D} \right)$$

At the singular point  $x_h = x_c$ ,

$$F(x_{c_0}, x_{R_0}, x_{c_0}) = 4 \frac{\sum_{i=1}^3 [A_i B + \frac{1}{2} B A] c_i}{\sqrt{A \left[ 4A \left( \frac{c}{D} \right)^2 - B^2 \right]}} \quad (103)$$

This known value at the singular point allows a straightforward analysis procedure to be undertaken using the procedures previously described.

Some convergence problems near the leading and trailing edges and over much of the surface for narrow blades (maximum  $c/D \approx 0.05$ ) have been resolved by computing the linearized form of  $F$  (Equation 100) over the entire blade and adding a correction term which is the difference between the actual integrand and this linear approximation. This option has been included in the computer program and is defined as "linear approximation-plus-difference." When conventional integration techniques are used everywhere except at the singular point, where Equation (103) is required, the procedure is defined as "direct."

For the trailing-vortex sheet, a regular integration can be performed since no singular points occur on the sheet. The strength of the vorticity is given by Equation (59) and the induced velocity field is given by

$$\frac{v}{V} = \frac{1}{4} \int_0^1 \left( - \frac{dG}{dx_R} \right) \sum_{b=1}^2 W(r_0, x_R, \theta_b) dx_R \quad (104)$$

where

$$W(r_0, x_R, \theta_b) = \int_0^{\infty} e_1 \times \frac{\frac{r_0}{D} \frac{w_b}{D}}{\left[ \frac{r_0}{D} \frac{w_b}{D} \right]^2} d\eta \quad (105)$$

( )

**Figure 1**

( )

$$d(\Gamma_1, \Gamma_2) = \max_{i \in \{1, \dots, n\}} |x_i - y_i|$$

$$\left( \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x) e^{-x^2} dx \right)^2 = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x)^2 e^{-x^2} dx$$

$$\cdot \left( \begin{array}{cc} & \\ D_1 & D_2 \end{array} \right) = \frac{1}{\sqrt{\pi}} e^{-x^2 + y^2}.$$

$$\left. \begin{aligned} & \sin \frac{1}{2} \pi \\ & \sin \frac{1}{2} \pi \end{aligned} \right\} = 0. \quad (11)$$

$$\frac{\cos \phi_p}{2} \left[ 2R - 2R_1 \cos (\theta_{1e} + \theta_h) \right]$$

$$+ 2\eta \frac{\cos \phi_p}{\sin \phi_p} \Big) + \left[ \left( \frac{v_{1c}}{v_{1e}} - \frac{v_{1c}}{v_{1e}} \right) \right]$$

$$\eta \sin \phi_p) \cos \phi_p \sin (\theta_{ir} + \theta_h - \phi$$

$$+ 2\eta \frac{\cos \phi_p}{\sin \phi_p} + \frac{\sin \phi_p}{\sin \phi_p} \left( x_p \right.$$

$$x_R \cos(\theta_{1r} + \theta_h - \phi + 2\eta \frac{\cos \phi_p}{k_p}) \Big] e$$

$$+ \left[ \left( \frac{x_{re}}{D} - r \sin \phi_p \right) \cos \phi_p \cos (\theta_{re} \right.$$

$$+ \theta_k = \varphi + 2\pi \left( \frac{\cos \phi_p}{\lambda_R} \right) + \frac{\sin \phi_p}{\lambda_R} \sin(\theta)$$

$$+ \left. \frac{\partial}{\partial \eta} \left( \frac{\cos \phi_p}{\eta} \right) \right] r_p(\phi) = 0 \quad (10a)$$

As a result, the model is able to capture the effects of the various factors on the dependent variable. The model is estimated using the following equation:

1.  $\frac{1}{2}$

1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 26

(iii)  $\mathcal{A} \in \mathcal{A}_1$  and  $\mathcal{B} \in \mathcal{A}_2$  are  $\mathcal{A}_1$ - and  $\mathcal{A}_2$ -invariant, respectively, and  
 (iv)  $\mathcal{A} \in \mathcal{A}_1$  and  $\mathcal{B} \in \mathcal{A}_2$  are  $\mathcal{A}_1$ - and  $\mathcal{A}_2$ -invariant, respectively, and

For the case of  $\alpha = 1$ , the following theorem holds:

There is a significant difference between the two groups ( $p < 0.05$ ).

et al., 1995; N. O. O. et al., 1996) and in some studies (N. O. O. et al., 1995; N. O. O. et al., 1996).

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for which the  $n+1$ st element of  $q$  is between successive points

1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 26

and the increment in angular variable  $\theta$  between successive points is

11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 841. 842. 843. 844. 845. 846. 847

1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 26

$$\Delta''_1 = \frac{r_1}{2} \frac{\cos 2p}{\sin p}$$

$$\Delta H_{12} = \frac{\Delta H_1}{\Delta H_1 + 1} - \frac{\Delta H_2}{\Delta H_2 + 1} \quad (47) \quad \Delta H_1$$

Consequently,  $\eta_1 = \eta_2$  and the equal increments of  $\chi/\eta$  are

$\eta_1 = 10$  has been satisfactory to date. When the distance between points becomes small, small time increments are

was determined by comparison with analytical results for two related but simpler problems that could be solved exactly.

general helical filament: all velocity components for a

accuracy to the third decimal point was found with the

of two units in the fourth decimal point was found.

In order to perform calculations, the form of  $y^*$  and two-dimensional thickness must also be specified. A general family of loading functions has been selected (10) with the property that they have zero values at the leading and trailing edges and resemble conventional NACA loading functions (11). The zero values at the ends are necessary for a finite fit with a sine series trigonometric interpolation polynomial. For loading distributions which approximate the NACA series meaning, the following "hordwise" forms are used:

$$y^* = \begin{cases} 0 & \text{for } x = 0 \\ \frac{1}{2} \left( \frac{x}{c} \right)^2 & \text{for } 0 < x < 1 \\ 0 & \text{for } x = 1 \end{cases} \quad (11a)$$

Fig. 1 Load distribution

The value of  $K$  must be taken sufficiently large to make the exact distribution in the leading edge region nearly tangent to a previous investigation (10) of this loading function for  $K = 8$  and  $10$  demonstrated that it was an acceptable approximation of the NACA airfoil's meanline (see Figure 2). Sine series "hordwise" loading functions were selected to

$$y^* = \sin^2 \left( \frac{\pi}{2} x \right) \quad \text{for } 0 < x < 1 \quad (11b)$$

Each one half chord distribution must be integrated across the chord and scaled to produce a unit value for the integral

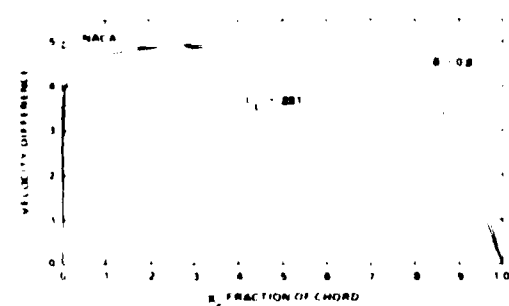


Fig. 1 Load distribution

The thickness offset is assumed in the form

$$\frac{t}{D} = \frac{(xR)^2}{D} + \frac{(xR)^4}{D} + Y_1(x) \quad (11c)$$

where the "hordwise" distribution,  $Y_1(x)$ , remains the same from root to tip; only the maximum value changes with radius. This is true for current propulsor designs. Specific examples of the thickness function included in the computer code are the NACA 4 and 5 digit sections (19), the NACA 16 section (20), an elliptic nose-quartic tail section similar to that described in Reference (21), and an approximate NACA airfoil (Model 22) section. All have been analytically defined.

## Computer Code—Convergence and Run Time

The complexity of the numerical analysis is such that error estimates are difficult to establish. A few loading cases exist for which analytic integrations may be performed, but comparisons do not usually evaluate the general case. The procedure selected to evaluate convergence was to vary the number of intervals in the radial direction,  $NR$ , and the number of intervals in the "hordwise" direction,  $NX$ . In addition, some radial and "hordwise" points were eliminated from the calculations. Computed values of the pitch and torque coefficients are shown in Table I together with a similar variation of data calculated according to the procedure described in Reference (7) for the same propulsor which is similar to NSRDC Model 4498. Computer input processing time is for computations at 13 radial between the extreme radii listed and is in seconds for the Burroughs 5700 High Speed Computer. Current charges are 3 cents per CPU second resulting in a maximum charge of about \$40. All procedures presented produce about equally satisfactory results with about only one percent difference in pitch and torque values about the same as found for Kerwin's numerical analysis. The computed pitch, however, is a few percent less than computed by Kerwin's method. Since limited flight experience at DINSRDC to date has been that Kerwin's procedure produces designs that are generally slightly overpitched, perhaps some improvement in performance may be expected using the present method.

Predictions of the pitch and torque by the two procedures developed for computing the induced velocity field on the blade surface which contains the field point—direct and approximate plus difference—are shown in Table I to be nearly the same. However, it has been found that overall the "approximate plus difference" procedures are preferable when dense chordwise spacing is chosen (e.g.,  $NX = 19$ ) or narrow blades (maximum  $t/D = 0.05$ ) are involved. In these situations, the "direct" procedure produces locally erratic values of the induced velocity because of the decreased spacing between adjacent lines of integration with a corresponding lack of accuracy in the numerical integrations for the resulting near-singular integrals. This effect is illustrated in Figure 3 which shows values of one of the helical components of the average induced velocity at the 0.946 radius of the reference blade. This velocity component is due to only loading on the blade itself; the effects of thickness, the other blades and the shed vortex sheet are not included. All data shown in subsequent figures have been computed by the "approximate plus difference" procedure although only the pressure distribution near the leading edge in the tip region of the blade was significantly different between the two procedures.

Overall run time varies with number of points, number of blades, and blade width. Since computer usage charges are so low, the 181 x 19 array size is recommended. For a narrow blade, the linear "approximation plus difference" procedure is recommended and the run time may increase by a few hundred seconds because of special care taken with the shed vortex sheet calculations. Computer execution time for Kerwin's program is unknown for the Burroughs 5700 high speed computer but is estimated to be about 150 seconds for data calculations at 4 chordwise points at 8 radial stations. For the results shown in Table I, data are computed at 13 radial stations with either 8, 11, or 17 chordwise points depending on input data specification.

Further details of the geometry of this example are given in Table II. Radial variables are titled according to the symbols suggested in Reference (10).

Table I  
Effect of Parameters on Pitch, Camber, and Computer Run Time

| NR                    | COMPUTATIONAL PROCEDURE |                    |                    |                   |                    |                          |                           | Kerwin Computation (7) |                      |
|-----------------------|-------------------------|--------------------|--------------------|-------------------|--------------------|--------------------------|---------------------------|------------------------|----------------------|
|                       | 181 x 10<br>Direct      | 181 x 10<br>Direct | 181 x 13<br>Direct | 91 x 19<br>Direct | 181 x 19<br>Direct | 91 x 19<br>Approx + Diff | 181 x 19<br>Approx + Diff | $\Delta\theta$ C M S   | $\Delta\theta$ C M S |
| PITCH DIAMETER        |                         |                    |                    |                   |                    |                          |                           |                        |                      |
| 4                     | 1.498                   | 1.498              | 1.51               | 1.53              | 1.538              | 1.539                    | 1.540                     | 1.580                  | 1.603                |
| 4                     | 1.466                   | 1.466              | 1.4                | 1.487             | 1.487              | 1.48                     | 1.488                     | 1.511                  | 1.527                |
| 4                     | 1.564                   | 1.564              | 1.577              | 1.577             | 1.577              | 1.577                    | 1.577                     | 1.592                  | 1.596                |
| 6.6                   | 1.184                   | 1.184              | 1.189              | 1.194             | 1.193              | 1.193                    | 1.193                     | 1.213                  |                      |
| 8                     | 1.038                   | 1.038              | 1.040              | 1.043             | 1.042              | 1.042                    | 1.042                     | 1.065                  | 1.070                |
| 14                    | 0.883                   | 0.883              | 0.882              | 0.884             | 0.883              | 0.883                    | 0.883                     | 0.894                  | 0.885                |
|                       |                         |                    |                    |                   |                    |                          |                           | 0.890                  |                      |
| CAMBER CHORD          |                         |                    |                    |                   |                    |                          |                           |                        |                      |
| 4                     | 0.0307                  | 0.0307             | 0.0315             | 0.0320            | 0.0327             | 0.0323                   | 0.0324                    | 0.0266                 | 0.0259               |
| 4                     | 0.0368                  | 0.0368             | 0.0367             | 0.0368            | 0.0368             | 0.0369                   | 0.0369                    | 0.0268                 |                      |
| 4                     | 0.0291                  | 0.0291             | 0.0293             | 0.0298            | 0.0295             | 0.0295                   | 0.0295                    | 0.0350                 | 0.0351               |
| 6.6                   | 0.0288                  | 0.0288             | 0.028              | 0.028             | 0.0258             | 0.0254                   | 0.0257                    | 0.0301                 | 0.0294               |
| 8                     | 0.0189                  | 0.0189             | 0.0189             | 0.0188            | 0.0189             | 0.0188                   | 0.0188                    | 0.0257                 |                      |
| 14                    | 0.0124                  | 0.0124             | 0.0122             | 0.0122            | 0.0123             | 0.0123                   | 0.0122                    | 0.0181                 | 0.0180               |
|                       |                         |                    |                    |                   |                    |                          |                           | 0.0122                 |                      |
|                       |                         |                    |                    |                   |                    |                          |                           | 0.0120                 | 0.0113               |
| Computer<br>Sec (min) | 4.00                    | 4.20               | 6.00               | 7.35              | 10.85              | 7.85                     | 11.35                     | N A                    | N A                  |



Fig. 3 Helical velocity component  $v_t$  vs  $x/V$

#### DISCUSSION OF EXAMPLE COMPUTATIONS

In this section, the consequences of choices the designer might make both for overall geometry and for the chordwise variation of the thickness distribution and loading distribution are examined. Some common variations in the location of the blade mid-chord line are investigated to determine the effects of overall geometry on pitch, camber, pressure distribution, and second-order performance coefficients. The variations are unskewed, skewed and warped blades with other input specifications the same. Skewed blades have blade sections displaced along the pitch helix and warped blades have blade sections displaced circumferentially in the plane at  $x = 0$ .

Input quantities and selected output are shown in Table II for a warped blade similar to NSRDC Model 4498 (and one similar to the example of Reference 7). For an unskewed blade, the column labeled TEIS, the skew angle  $\theta_s$ , would be zero, and for a skewed blade the column labeled RAKE/D, the total rake  $\alpha_1/D$ , would be equal to  $P/\theta_s$  ( $2\pi/D$ ). In Figure 4, the computed pitch and camber ratios are shown for these various overall geometries with all other input the same as in Table II. In Figure 5, the effects of the chordwise load distribution and chordwise thickness function on pitch and camber are shown. The effect of rake and skew on pitch and camber follows known trends (7, 24). The effect of thickness distribution on pitch and camber is negligible and the effect of elliptic loading is to reduce the pitch and increase the camber, as would be the case in two dimensional flow at the ideal angle for a given lift coefficient. In Figure 6, the pitch and camber change is shown for another modification of the warped blade. Since a large change occurs in the pitch from the input specification (Table II) to the computed values (Figure 4), computations were performed with the singularities distributed on the blade reference surface at a pitch taken from Figure 4. This change in pitch places the singularities nearer the final blade surface. To have uniformity in the calculations, the pitch angle of the shed vortex sheet was taken as  $\beta$ , the advance angle of the shed vortex sheet. (In Figure 4, the shed vortex sheet was taken to be at the input pitch, which is  $\beta_1$ , the pitch angle derived from the solution of a straight radial lifting line representing each blade.) The change in pitch angle of the shed vortex wake from  $\beta_1$  to  $\beta$  produces a slight increase of pitch near the hub (compare data in Figures 4 and 6). A change in the pitch of the blade reference surface to the values shown by the dashed curve in Figure 4 produces a significant reduction in computed pitch and a compensating increase in camber near the root, with negligible change in either pitch or camber from about  $x/R = 0.5$  to the tip. Hence the orientation of the free vortex sheet and blade reference surface have significant effects on the pitch and camber values only near the hub.

Table II  
DEFINITION OF DESIGN EXAMPLE  
Sample Data from Computer Code

6698 EXAMPLE LOADING AND THICKNESS 5 BLADES - SEPT 80

CIRCULATION COEFFICIENTS

| N | G(N)      |
|---|-----------|
| 1 | 0.029028  |
| 2 | 0.007010  |
| 3 | -0.002810 |
| 4 | -0.000560 |
| 5 | 0.000081  |
| 6 | 0.000081  |

DIAMETER = DP C.1048 R ADVEN V(FEND) = 0.9880 NPB = 2 = 5

INPUT DATA

| IN      | CM/DP   | PP/DP   | RARE/DP | TETS    | THAR/CH |
|---------|---------|---------|---------|---------|---------|
| 0.20000 | 0.16500 | 1.16270 | 0.00000 | 0.00000 | 0.24000 |
| 0.25000 | 0.19700 | 1.17510 | 0.00000 | 0.07854 | 0.19800 |
| 0.30000 | 0.22900 | 1.18750 | 0.00000 | 0.15708 | 0.15610 |
| 0.35000 | 0.27500 | 1.19980 | 0.00000 | 0.23562 | 0.11400 |
| 0.40000 | 0.31200 | 1.21220 | 0.00000 | 0.31416 | 0.07190 |
| 0.45000 | 0.33700 | 1.22460 | 0.00000 | 0.39270 | 0.02980 |
| 0.50000 | 0.36700 | 1.23700 | 0.00000 | 0.47124 | 0.00790 |
| 0.55000 | 0.39400 | 1.24940 | 0.00000 | 0.54978 | 0.00590 |
| 0.60000 | 0.41600 | 1.26180 | 0.00000 | 0.62832 | 0.00390 |
| 0.65000 | 0.43400 | 1.27420 | 0.00000 | 0.70686 | 0.00190 |
| 0.70000 | 0.44800 | 1.28660 | 0.00000 | 0.78540 | 0.00090 |
| 0.75000 | 0.45800 | 1.29900 | 0.00000 | 0.86394 | 0.00040 |
| 0.80000 | 0.46400 | 1.31140 | 0.00000 | 0.94248 | 0.00020 |
| 0.85000 | 0.46700 | 1.32380 | 0.00000 | 1.02102 | 0.00010 |
| 0.90000 | 0.46800 | 1.33620 | 0.00000 | 1.10000 | 0.00000 |
| 1.00000 | 0.46800 | 1.34860 | 0.00000 | 1.17854 | 0.00000 |

6698 EXAMPLE LOADING AND THICKNESS 5 BLADES - SEPT 80

| IN      | CM/DP   | (CM/DP) <sup>2</sup> | PP/DP   | (PP/DP) <sup>2</sup> | RARE/DP | TETS    | (TETS) <sup>2</sup> | THAR/CH | (THAR/DP) | W/F     |
|---------|---------|----------------------|---------|----------------------|---------|---------|---------------------|---------|-----------|---------|
| 0.20000 | 0.16500 | 0.027225             | 1.16270 | 1.35086              | 0.00000 | 0.00000 | 0.00000             | 0.24000 | -0.02619  | 1.00000 |
| 0.25000 | 0.19700 | 0.038809             | 1.17510 | 1.38086              | 0.00000 | 0.07854 | 0.00618             | 0.19800 | -0.02619  | 1.00000 |
| 0.30000 | 0.22900 | 0.052441             | 1.18750 | 1.40800              | 0.00000 | 0.15708 | 0.02475             | 0.15610 | -0.02619  | 1.00000 |
| 0.35000 | 0.27500 | 0.075625             | 1.19980 | 1.43952              | 0.00000 | 0.23562 | 0.05571             | 0.11400 | -0.02619  | 1.00000 |
| 0.40000 | 0.31200 | 0.097344             | 1.21220 | 1.46942              | 0.00000 | 0.31416 | 0.09866             | 0.07190 | -0.02619  | 1.00000 |
| 0.45000 | 0.33700 | 0.113569             | 1.22460 | 1.50062              | 0.00000 | 0.39270 | 0.15421             | 0.02980 | -0.02619  | 1.00000 |
| 0.50000 | 0.36700 | 0.134689             | 1.23700 | 1.53202              | 0.00000 | 0.47124 | 0.22204             | 0.00790 | -0.02619  | 1.00000 |
| 0.55000 | 0.39400 | 0.155236             | 1.24940 | 1.56362              | 0.00000 | 0.54978 | 0.30228             | 0.00590 | -0.02619  | 1.00000 |
| 0.60000 | 0.41600 | 0.173056             | 1.26180 | 1.59542              | 0.00000 | 0.62832 | 0.39478             | 0.00390 | -0.02619  | 1.00000 |
| 0.65000 | 0.43400 | 0.188356             | 1.27420 | 1.62742              | 0.00000 | 0.70686 | 0.50078             | 0.00190 | -0.02619  | 1.00000 |
| 0.70000 | 0.44800 | 0.200704             | 1.28660 | 1.65962              | 0.00000 | 0.78540 | 0.61664             | 0.00090 | -0.02619  | 1.00000 |
| 0.75000 | 0.45800 | 0.209644             | 1.29900 | 1.69202              | 0.00000 | 0.86394 | 0.74838             | 0.00040 | -0.02619  | 1.00000 |
| 0.80000 | 0.46400 | 0.215296             | 1.31140 | 1.72462              | 0.00000 | 0.94248 | 0.88832             | 0.00020 | -0.02619  | 1.00000 |
| 0.85000 | 0.46700 | 0.217969             | 1.32380 | 1.75742              | 0.00000 | 1.02102 | 1.04248             | 0.00010 | -0.02619  | 1.00000 |
| 0.90000 | 0.46800 | 0.219024             | 1.33620 | 1.79042              | 0.00000 | 1.10000 | 1.21000             | 0.00000 | -0.02619  | 1.00000 |
| 0.95000 | 0.46800 | 0.219024             | 1.34860 | 1.82362              | 0.00000 | 1.17854 | 1.39078             | 0.00000 | -0.02619  | 1.00000 |
| 1.00000 | 0.46800 | 0.219024             | 1.36100 | 1.85702              | 0.00000 | 1.25708 | 1.58042             | 0.00000 | -0.02619  | 1.00000 |

6698 EXAMPLE LOADING AND THICKNESS 5 BLADES - SEPT 80

ELLIPSE WITH AIRFOIL TAIL AND CROWNED LEAD

| EL      | ST      | DT/DANG | STANG   | COBANG  | ZO U/F  | GBN +   | GBNBT   | W/F     |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.00750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.01500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.02250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.03000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.03750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.04500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.05250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.06000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.06750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.07500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.08250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.09000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.09750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.10500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.11250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.12000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.12750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.13500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.14250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.15000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.15750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.16500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.17250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.18000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.18750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.19500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.20250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.21000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.21750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.22500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.23250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.24000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.24750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.25500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.26250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.27000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.27750 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.28500 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.29250 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |
| 0.30000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 1.00000 |

IDEAL ANGLE = 2.000 DEG

○ ○ ○ ○ ○

| Case | Time  | Lat   | Long   | Alt  | Mag | Filter | Notes |
|------|-------|-------|--------|------|-----|--------|-------|
| 1    | 13:00 | 14.00 | 107.00 | 1000 | 1.5 | B      |       |
| 2    | 13:01 | 14.01 | 107.01 | 1000 | 1.5 | B      |       |
| 3    | 13:02 | 14.02 | 107.02 | 1000 | 1.5 | B      |       |
| 4    | 13:03 | 14.03 | 107.03 | 1000 | 1.5 | B      |       |
| 5    | 13:04 | 14.04 | 107.04 | 1000 | 1.5 | B      |       |
| 6    | 13:05 | 14.05 | 107.05 | 1000 | 1.5 | B      |       |
| 7    | 13:06 | 14.06 | 107.06 | 1000 | 1.5 | B      |       |
| 8    | 13:07 | 14.07 | 107.07 | 1000 | 1.5 | B      |       |
| 9    | 13:08 | 14.08 | 107.08 | 1000 | 1.5 | B      |       |
| 10   | 13:09 | 14.09 | 107.09 | 1000 | 1.5 | B      |       |
| 11   | 13:10 | 14.10 | 107.10 | 1000 | 1.5 | B      |       |
| 12   | 13:11 | 14.11 | 107.11 | 1000 | 1.5 | B      |       |
| 13   | 13:12 | 14.12 | 107.12 | 1000 | 1.5 | B      |       |
| 14   | 13:13 | 14.13 | 107.13 | 1000 | 1.5 | B      |       |
| 15   | 13:14 | 14.14 | 107.14 | 1000 | 1.5 | B      |       |
| 16   | 13:15 | 14.15 | 107.15 | 1000 | 1.5 | B      |       |
| 17   | 13:16 | 14.16 | 107.16 | 1000 | 1.5 | B      |       |
| 18   | 13:17 | 14.17 | 107.17 | 1000 | 1.5 | B      |       |
| 19   | 13:18 | 14.18 | 107.18 | 1000 | 1.5 | B      |       |
| 20   | 13:19 | 14.19 | 107.19 | 1000 | 1.5 | B      |       |
| 21   | 13:20 | 14.20 | 107.20 | 1000 | 1.5 | B      |       |
| 22   | 13:21 | 14.21 | 107.21 | 1000 | 1.5 | B      |       |
| 23   | 13:22 | 14.22 | 107.22 | 1000 | 1.5 | B      |       |
| 24   | 13:23 | 14.23 | 107.23 | 1000 | 1.5 | B      |       |
| 25   | 13:24 | 14.24 | 107.24 | 1000 | 1.5 | B      |       |
| 26   | 13:25 | 14.25 | 107.25 | 1000 | 1.5 | B      |       |
| 27   | 13:26 | 14.26 | 107.26 | 1000 | 1.5 | B      |       |
| 28   | 13:27 | 14.27 | 107.27 | 1000 | 1.5 | B      |       |
| 29   | 13:28 | 14.28 | 107.28 | 1000 | 1.5 | B      |       |
| 30   | 13:29 | 14.29 | 107.29 | 1000 | 1.5 | B      |       |
| 31   | 13:30 | 14.30 | 107.30 | 1000 | 1.5 | B      |       |
| 32   | 13:31 | 14.31 | 107.31 | 1000 | 1.5 | B      |       |
| 33   | 13:32 | 14.32 | 107.32 | 1000 | 1.5 | B      |       |
| 34   | 13:33 | 14.33 | 107.33 | 1000 | 1.5 | B      |       |
| 35   | 13:34 | 14.34 | 107.34 | 1000 | 1.5 | B      |       |
| 36   | 13:35 | 14.35 | 107.35 | 1000 | 1.5 | B      |       |
| 37   | 13:36 | 14.36 | 107.36 | 1000 | 1.5 | B      |       |
| 38   | 13:37 | 14.37 | 107.37 | 1000 | 1.5 | B      |       |
| 39   | 13:38 | 14.38 | 107.38 | 1000 | 1.5 | B      |       |
| 40   | 13:39 | 14.39 | 107.39 | 1000 | 1.5 | B      |       |
| 41   | 13:40 | 14.40 | 107.40 | 1000 | 1.5 | B      |       |
| 42   | 13:41 | 14.41 | 107.41 | 1000 | 1.5 | B      |       |
| 43   | 13:42 | 14.42 | 107.42 | 1000 | 1.5 | B      |       |
| 44   | 13:43 | 14.43 | 107.43 | 1000 | 1.5 | B      |       |
| 45   | 13:44 | 14.44 | 107.44 | 1000 | 1.5 | B      |       |
| 46   | 13:45 | 14.45 | 107.45 | 1000 | 1.5 | B      |       |
| 47   | 13:46 | 14.46 | 107.46 | 1000 | 1.5 | B      |       |
| 48   | 13:47 | 14.47 | 107.47 | 1000 | 1.5 | B      |       |
| 49   | 13:48 | 14.48 | 107.48 | 1000 | 1.5 | B      |       |
| 50   | 13:49 | 14.49 | 107.49 | 1000 | 1.5 | B      |       |
| 51   | 13:50 | 14.50 | 107.50 | 1000 | 1.5 | B      |       |
| 52   | 13:51 | 14.51 | 107.51 | 1000 | 1.5 | B      |       |
| 53   | 13:52 | 14.52 | 107.52 | 1000 | 1.5 | B      |       |
| 54   | 13:53 | 14.53 | 107.53 | 1000 | 1.5 | B      |       |
| 55   | 13:54 | 14.54 | 107.54 | 1000 | 1.5 | B      |       |
| 56   | 13:55 | 14.55 | 107.55 | 1000 | 1.5 | B      |       |
| 57   | 13:56 | 14.56 | 107.56 | 1000 | 1.5 | B      |       |
| 58   | 13:57 | 14.57 | 107.57 | 1000 | 1.5 | B      |       |
| 59   | 13:58 | 14.58 | 107.58 | 1000 | 1.5 | B      |       |
| 60   | 13:59 | 14.59 | 107.59 | 1000 | 1.5 | B      |       |
| 61   | 14:00 | 15.00 | 108.00 | 1000 | 1.5 | B      |       |
| 62   | 14:01 | 15.01 | 108.01 | 1000 | 1.5 | B      |       |
| 63   | 14:02 | 15.02 | 108.02 | 1000 | 1.5 | B      |       |
| 64   | 14:03 | 15.03 | 108.03 | 1000 | 1.5 | B      |       |
| 65   | 14:04 | 15.04 | 108.04 | 1000 | 1.5 | B      |       |
| 66   | 14:05 | 15.05 | 108.05 | 1000 | 1.5 | B      |       |
| 67   | 14:06 | 15.06 | 108.06 | 1000 | 1.5 | B      |       |
| 68   | 14:07 | 15.07 | 108.07 | 1000 | 1.5 | B      |       |
| 69   | 14:08 | 15.08 | 108.08 | 1000 | 1.5 | B      |       |
| 70   | 14:09 | 15.09 | 108.09 | 1000 | 1.5 | B      |       |
| 71   | 14:10 | 15.10 | 108.10 | 1000 | 1.5 | B      |       |
| 72   | 14:11 | 15.11 | 108.11 | 1000 | 1.5 | B      |       |
| 73   | 14:12 | 15.12 | 108.12 | 1000 | 1.5 | B      |       |
| 74   | 14:13 | 15.13 | 108.13 | 1000 | 1.5 | B      |       |
| 75   | 14:14 | 15.14 | 108.14 | 1000 | 1.5 | B      |       |
| 76   | 14:15 | 15.15 | 108.15 | 1000 | 1.5 | B      |       |
| 77   | 14:16 | 15.16 | 108.16 | 1000 | 1.5 | B      |       |
| 78   | 14:17 | 15.17 | 108.17 | 1000 | 1.5 | B      |       |
| 79   | 14:18 | 15.18 | 108.18 | 1000 | 1.5 | B      |       |
| 80   | 14:19 | 15.19 | 108.19 | 1000 | 1.5 | B      |       |
| 81   | 14:20 | 15.20 | 108.20 | 1000 | 1.5 | B      |       |
| 82   | 14:21 | 15.21 | 108.21 | 1000 | 1.5 | B      |       |
| 83   | 14:22 | 15.22 | 108.22 | 1000 | 1.5 | B      |       |
| 84   | 14:23 | 15.23 | 108.23 | 1000 | 1.5 | B      |       |
| 85   | 14:24 | 15.24 | 108.24 | 1000 | 1.5 | B      |       |
| 86   | 14:25 | 15.25 | 108.25 | 1000 | 1.5 | B      |       |
| 87   | 14:26 | 15.26 | 108.26 | 1000 | 1.5 | B      |       |
| 88   | 14:27 | 15.27 | 108.27 | 1000 | 1.5 | B      |       |
| 89   | 14:28 | 15.28 | 108.28 | 1000 | 1.5 | B      |       |
| 90   | 14:29 | 15.29 | 108.29 | 1000 | 1.5 | B      |       |
| 91   | 14:30 | 15.30 | 108.30 | 1000 | 1.5 | B      |       |
| 92   | 14:31 | 15.31 | 108.31 | 1000 | 1.5 | B      |       |
| 93   | 14:32 | 15.32 | 108.32 | 1000 | 1.5 | B      |       |
| 94   | 14:33 | 15.33 | 108.33 | 1000 | 1.5 | B      |       |
| 95   | 14:34 | 15.34 | 108.34 | 1000 | 1.5 | B      |       |
| 96   | 14:35 | 15.35 | 108.35 | 1000 | 1.5 | B      |       |
| 97   | 14:36 | 15.36 | 108.36 | 1000 | 1.5 | B      |       |
| 98   | 14:37 | 15.37 | 108.37 | 1000 | 1.5 | B      |       |
| 99   | 14:38 | 15.38 | 108.38 | 1000 | 1.5 | B      |       |
| 100  | 14:39 | 15.39 | 108.39 | 1000 | 1.5 | B      |       |

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in financial matters.

2. The second part outlines the various methods and tools used to collect and analyze data. This includes both traditional manual methods and modern digital technologies, highlighting the advantages of each approach.

3. The third part focuses on the challenges faced in data collection and analysis, such as incomplete data, biases, and the need for standardized protocols to ensure consistency across different studies.

4. The fourth part discusses the ethical considerations surrounding data collection and analysis, including issues related to privacy, informed consent, and the potential for misuse of data.

5. The fifth part provides a summary of the key findings and conclusions drawn from the research, emphasizing the need for continued research and innovation in data management and analysis.

6. The final part of the document offers recommendations for future research and practical applications of the findings, suggesting ways to improve data collection and analysis processes in various fields.

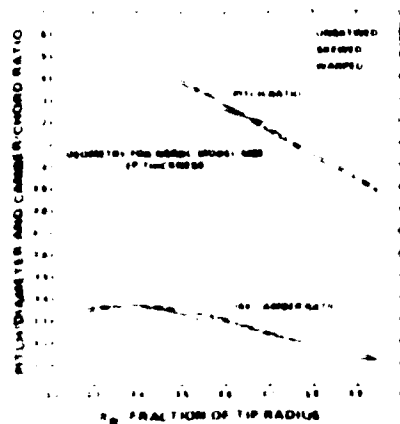
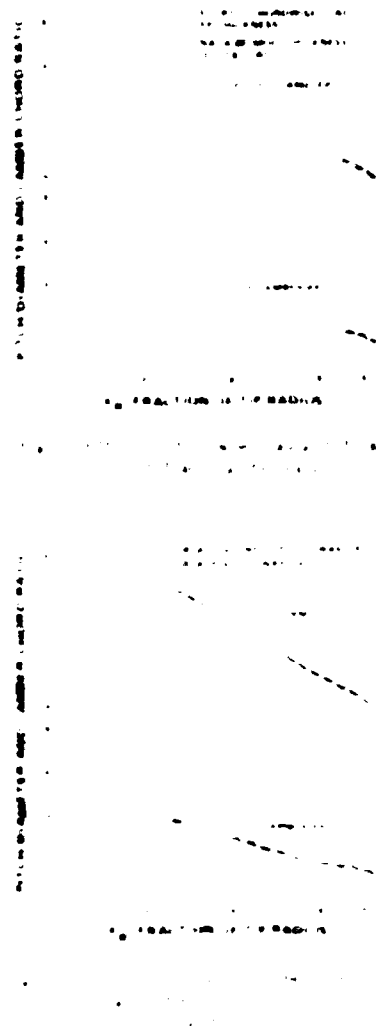


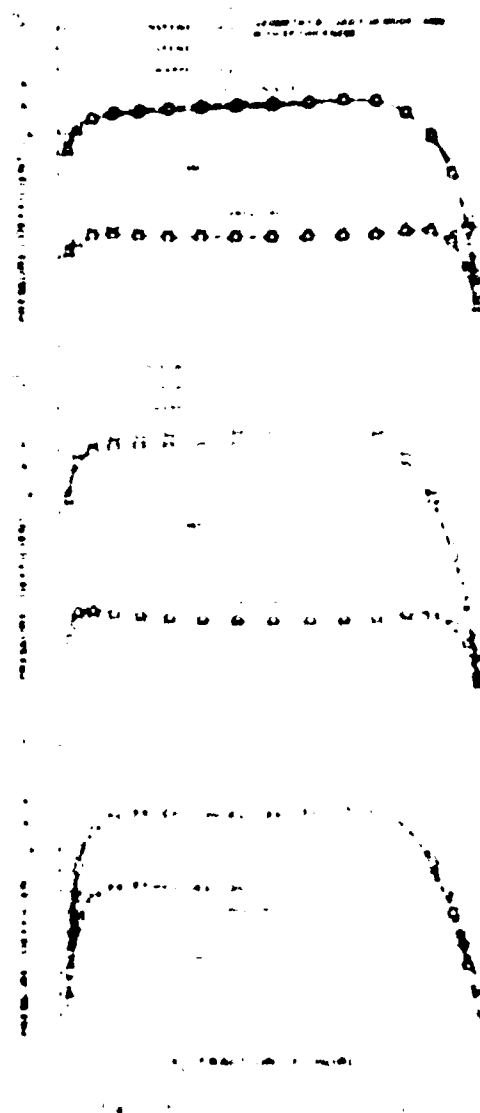
Fig. 4. Effect of skew and rake on lift coefficient values.

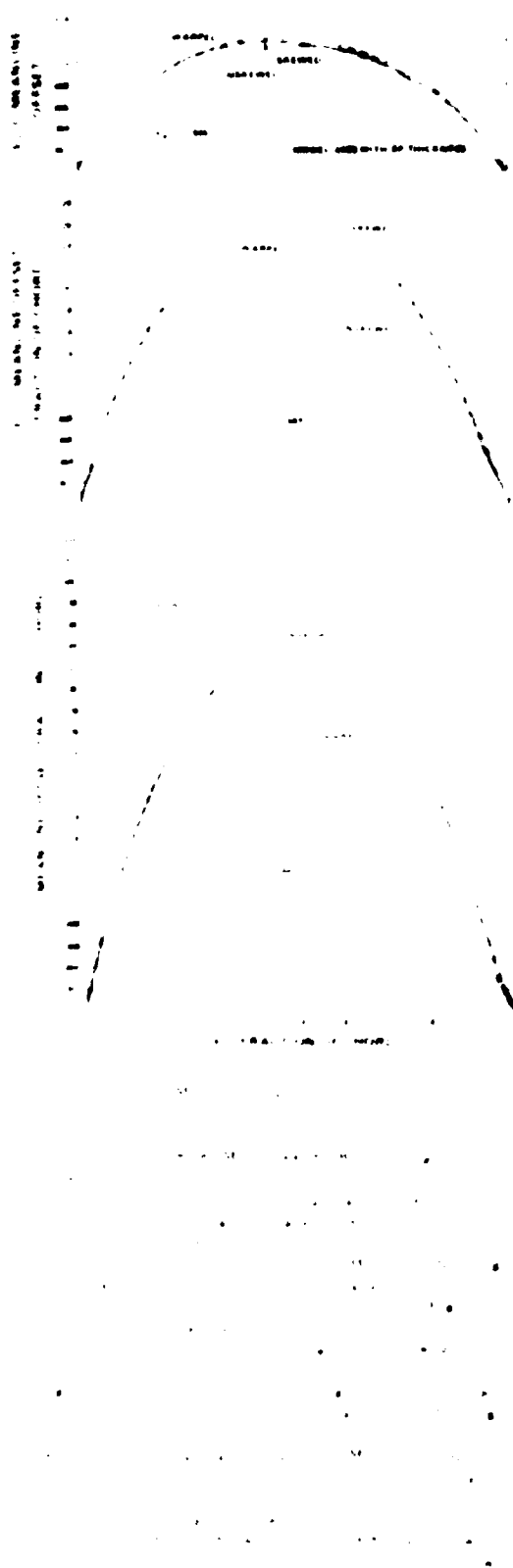


In Figure 5, blade pressure coefficient distribution on the warped, skewed and unskewed blade are shown for three radii: one near the hub, one near mid span, and one near the tip. The major difference in pressure distribution occurs near the hub, where the warped blade has greater suction on both sides of the blade, and hence a greater tendency to cavitate when the local pressure reaches the vapor pressure.

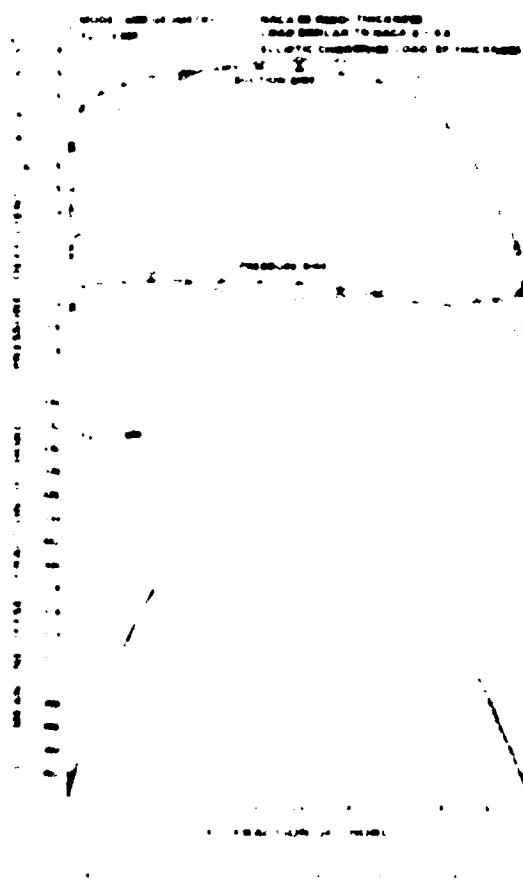
In Figure 6, the meanline shapes for the three blades at the same three radii are shown. The greatest change in meanline shape occurs at the root but is slight at the tip of the blade.

In Figure 7, the meanline cross section distribution is shown. Equations (1) and (2) and meanline shapes are shown at  $x/R = 0.001$  for the same variation in  $r/R$  that was used in the pressure distributions shown in Figure 5. The results are





modifications. The performance coefficients computed according to the lifting-line model and first-order linear lifting-surface model (Eqs. 9 and Equations 16a and 17a) and  $\Delta C_p$  from Equation 14a are nearly identical. The non-linear performance coefficients of #0 in Equations 16a and 17a and  $C_p$  from Equation 16b for a blade with 10% loading ( $C_{p0} = 0.1$ ) are increased a few percent relative to the lifting-line values. The addition of thickness and skew or warp changes the pressure distribution and the mean slope, and it raises the values by another few percent to a level barely producing values of  $C_{p0}$  and  $C_p$  which are more than ten percent greater than predicted by the lifting-line model. In all, the pressure distribution and force and moment coefficients and order of magnitude have been calculated and introduced with four percent error. It is well known that the associated third-order theory is not order 1/3 as accurate as values of order 1/2. Experimental evaluation is required to confirm the predictions. Predictions given in Table III should be interpreted as possible trends in the actual performance. The present lifting-line model, improved for proper design, is 1/4 as accurate as a rough estimate with a missing 100% height to represent the blade and a further 50% expected to be an acceptable estimate of the actual lifting ability. Similarly, the 1/4 model of warped blades is 1/4 as accurate, the majority of applications have large airfoil sections with a few percent of the lift, and with the simple lifting-line the significant differences have also been noted. The present curved lifting-line model requires a more lifting-line performance prediction, which should be more accurately calculated.







When the program is run on rotating blades, the program will produce the output from the computer and will also calculate the pressure distribution on the surface of the blades (see Appendix I for details of the computations).

#### NOTES AND REMARKS

1. The program is designed for determining the pressure distribution corresponding to a prescribed velocity distribution. The computer run times are as great as 10 minutes for the DINSRDC Burroughs 5700 high-speed computer. The velocity distributions are assumed to be uniform and pitch distributions are assumed to be constant and chordwise distributions are assumed to be constant. A radial velocity distribution is also possible. A radial velocity distribution is assumed to be constant and chordwise distributions are assumed to be constant.

2. The program is dependent upon the velocity distribution. The velocity distribution is assumed to be constant and chordwise distributions are assumed to be constant. The program is dependent upon the velocity distribution. The velocity distribution is assumed to be constant and chordwise distributions are assumed to be constant.

3. The program is dependent upon the velocity distribution. The velocity distribution is assumed to be constant and chordwise distributions are assumed to be constant. The program is dependent upon the velocity distribution. The velocity distribution is assumed to be constant and chordwise distributions are assumed to be constant.

#### REFERENCES

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#### APPENDIX STREAMLINE COORDINATE SYSTEM

It is often convenient to have an orthogonal coordinate system on the surface of the blade. In particular, for performing boundary-layer computations, an orthogonal coordinate system with one variable along the streamlines reduces the number of terms in the governing equations. To determine the differential equation of the streamline path, let

$$x_R = \psi(x_c) \quad (120)$$

be the radius of the streamlines as a function of the chordwise coordinate  $x_c$ . Then

$$s^*(x_c) = s(x_c, \psi(x_c)) \quad (121)$$

is the position vector of the streamlines on the blade surface. Hence a tangent to the streamline is

$$\begin{aligned} t_v &= \frac{ds^*}{dx_c} = \left( \frac{\partial s}{\partial x_c} \right)_{x_R=\psi} + \left( \frac{\partial s}{\partial x_R} \right)_{x_c=\psi} \frac{d\psi}{dx_R} \\ &= D \left[ \frac{c}{D} e_1 + \left( \alpha e_1 + \frac{N_{R0}^2 c}{D^2} \frac{c}{D} \right) \frac{d\psi}{dx_R} \right] \quad (122) \\ &= D \left[ \left( \frac{c}{D} + \alpha \frac{d\psi}{dx_c} \right) e_1 + \frac{1}{2} \sqrt{1+N_{R0}^2} \frac{d\psi}{dx_c} e \right] \end{aligned}$$

For this tangent vector to be parallel to the velocity vector on the surface, the vector cross product,  $t_v \times q$ , must be zero. Hence, for the velocity on the blade surface given by

$$\begin{aligned} \frac{q}{V} &= \frac{q_\alpha}{V} + \frac{v}{V} \\ &= \frac{U}{V} e_1 + \frac{W}{V} e \quad (123) \end{aligned}$$

the cross product is

$$\begin{aligned} \frac{t_v}{D} \times \frac{q}{V} &= \left\{ \left( \frac{c}{D} + \alpha \frac{d\psi}{dx_c} \right) e_1 + \frac{\sqrt{1+N_{R0}^2}}{2} \frac{d\psi}{dx_c} e \right\} \\ &\quad \times \left\{ \frac{U}{V} e_1 + \frac{W}{V} e \right\} \end{aligned}$$

$$\left\{ \frac{1}{2} \sqrt{1+N_{R0}^2} \frac{d\psi}{dx_c} e_1 + \left( \frac{c}{D} + \alpha \frac{d\psi}{dx_c} \right) \frac{W}{V} e \right\}$$

For this cross product to be zero, the slope of the streamline is

$$\frac{d\psi}{dx_c} = - \frac{\frac{c}{D} \frac{W}{V}}{\frac{1}{2} \sqrt{1+N_{R0}^2} \frac{U}{V} + \alpha \frac{W}{V}} \quad (124)$$

For lines along the surface which are normal to the streamlines, let

$$x_c = \kappa(x_R) \quad (125)$$

be the chordwise position as a function of radius. Then a vector on the blade surface tangent to this line is

$$\begin{aligned} t_n &= \frac{ds(\kappa(x_R), x_R)}{dx_R} \\ &= \left( \frac{\partial s}{\partial x_c} \right)_{x_R=\kappa} \frac{d\kappa}{dx_R} + \left( \frac{\partial s}{\partial x_R} \right)_{x_c=\kappa} \\ &= D \left[ \frac{c}{D} e_1 \frac{d\kappa}{dx_R} + \alpha e_1 + \frac{1}{2} \sqrt{1+N_{R0}^2} e \right] \quad (126) \end{aligned}$$

The condition to be satisfied is that  $t_n$  be perpendicular to the velocity vector, or

$$\begin{aligned} \frac{t_n}{D} \cdot \frac{q}{V} &= 0 \quad (127) \\ &= \frac{U}{V} \left( \frac{c}{D} \frac{d\kappa}{dx_R} + \alpha \right) + \frac{1}{2} \sqrt{1+N_{R0}^2} \frac{W}{V} \quad (128) \end{aligned}$$

Thus the slope of lines on the surface which are normal to the streamline is

$$\frac{d\kappa}{dx_R} = - \frac{\frac{1}{2} \sqrt{1+N_{R0}^2} \frac{W}{V} + \alpha \frac{U}{V}}{\frac{c}{D} \frac{U}{V}} \quad (129)$$

One now has differential equations to determine an orthogonal network over the blade surface. The differential arc length along the streamlines is

$$ds = \left\{ \left( \frac{\partial s}{\partial x_c} \right)_{x_R=\psi} + \left( \frac{\partial s}{\partial x_R} \right)_{x_c=\psi} \frac{d\psi}{dx_c} \right\} dx_c \quad (130)$$

Since

$$ds = |ds| = h_1 dx_c \quad (131)$$

then

$$h_1 = \left| \left( \frac{\partial s}{\partial x_c} \right)_{x_R = \psi} + \left( \frac{\partial s}{\partial x_R} \right)_{x_R = \psi} \frac{d\psi}{dx_c} \right|$$

or

$$\frac{h_1}{D} = \left\{ \left( \frac{\gamma}{D} + \alpha \frac{d\psi}{dx_c} \right)^2 + \frac{1 + N_{R0}^2}{4} \left( \frac{d\psi}{dx_c} \right)^2 \right\}^{1/2} \quad (132)$$

Similarly the differential arc length along the orthogonal surface coordinate is

$$ds = \left| \left( \frac{\partial s}{\partial x_c} \right)_{x_c = \kappa} \frac{d\kappa}{dx_R} + \left( \frac{\partial s}{\partial x_R} \right)_{x_c = \kappa} \right| dx_R \quad (133)$$

$$h_2 dx_R \quad (134)$$

where

$$\frac{h_2}{D} = \left\{ \left( \frac{\gamma}{D} \frac{d\kappa}{dx_R} + \alpha \right)^2 + \frac{1 + N_{R0}^2}{4} \right\}^{1/2} \quad (135)$$

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